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Operating Point Shift Induced by Relay Asymmetry: An Iterative Solution Proposal

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Abstract—**This contribution focuses on a problem that appears when using a relay with non-symmetric output in the closed loop. Such a scheme is usually used for process model parameters identification, possibly followed by automatic controller tuning. Whenever static or dynamic properties of the process reveal asymmetry when the sign of the input changes, the setpoint (reference) becomes different from the operating point value of the process output. As a class of relay-based identification methods utilize calculations in the frequency domain that are based on integral computation around the operating point, the discrepancy between the setpoint and the operating point can lead to incorrect results. The aim of the paper is mainly to provide the reader with problem formulation and step-by-step proposition of how it can be solved. Concise numerical examples are also given. The concluding remarks suggest possible further ways of research.**

Keywords—asymmetry, identification, relay, iterations, optimization.

I. Introduction

Most mathematical models of dynamical systems assume that both process static and dynamic properties remain independent with respect to the relative change of input variable sign (related to the operating point). These systems have a favorable property that the operation point remains equal to the reference when the feedback loop is closed. Hence, no one has to care about a discrepancy between the current setpoint value and the "true" process output operating point when identification or control.

However, the situation changes whenever the process evinces asymmetric gain or diverse time constants in its dynamics, depending on the input. It is worth noting that we do not primarily assume process nonlinearity; in this paper, the model is considered linear in the vicinity of the operating point. Processes with mass and energy transfer are epitomes of these asymmetric systems [1]. However, they can also be observed in other fields of human activity. For instance, in microelectronics [2], quantum physics [3], or economy [4].

This paper is focused on the use of the relay (or, generally, another simple non-linear element) in the closed loop in order to estimate process model parameters [5], [6]. The cornerstone of this family of parameter identification methods is to reach sustained oscillations around the operating point that are further analyzed and processed via several mathematical principles. It is apparent from the above-given introduction that the asymmetric dynamics requires the development of a special framework concept.

Two research questions arise herein. First, one has to be able to assemble two different submodels (more precisely, their parameter sets) from the oscillation data. As both relative positive and negative inputs and outputs with respect to the setpoint and operating point are reached, the data includes information about the behavior of both asymmetric submodels. Second, if an asymmetric relay is set, a mismatch between the reference and operating point appears (as indicated above) and requires to estimate the "true" operating point value. This value can be intuitively and roughly defined as follows. Let for the given process, estimated operating point, and obtained sustained oscillations, the computed process static gain [6], [7] be equal to the factual static gain. Then the operation point, based on which the static gain is computed, is the "true" one. It is naturally expected that the model parameters representing its dynamics can theoretically be found exactly for this point as well.

The aim of this contribution is, hence, twofold. We attempt to provide the reader with an idea of how to guess the intrinsic operating point. Simultaneously, based on this information, an iterative procedure of two asymmetric submodels estimation based on the asymmetric relay-feedback experiment is proposed. Two numerical examples are given in the paper to elucidate the motivation and ideas for a broader audience and maximum clarity. Several propositions and opening ideas of how the concept can be improved and what research can be done in the future conclude this contribution.

It is worth noting that preliminary concise and rigorous formulations of the presented thoughts together with a rich comparative study have recently been submitted to a journal [8]. Nevertheless, due to limited space, a detailed motivation and a step-by-step train of thought regarding the proposed concept have not been made.

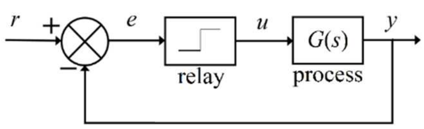

Fig. 1. A sketch of the basic relay-feedback system for identification.

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II. RELAY FEEDBACK FOR IDENTIFICATION

In this section, a framework principle of the model parameter identification based on the relay-feedback test is introduced. The emphasis is put on the asymmetric relay static characteristics. For simplicity, the very basic concept is presented avoiding advanced techniques that, e.g., make the estimation more accurate, save experimental time, reduce the number of feedback tests, etc. Then, a particular frequency-based method is sketched that is used for further simulation experiments in this paper. The reader is referred to literature resources and references therein, see, e.g., [5]-[7].

If the process can be stabilized by the selected relay, periodic courses of $u(t), y(t)$ (called the sustained oscillations) with angular frequency $\omega_{osc} = 2\pi / T_{osc}$ can be observed after a while. The advantage is that the signal values remain in the vicinity of the operating point, which is desirable when industrial applications [5].

The sustained oscillations can further be processed and evaluated. In principle, three main classes of relay-based identification methods exist [6]. Namely, first, the time-domain curve fitting methods attempt to match the measured responses [9]. Second, describing function approaches utilize a linearized input-output relay relationship based on the Fourier series expansion [5], [10]. Third, the curve fitting is made in the frequency domain by matching multiple frequency-response points [7], [11], [12]. The last family of methods is the most important for the sake of this paper since the parameter estimation accuracy of such techniques fundamentally depends on the operating point guess.

A. *Selected Method in the Frequency Domain*

Considerthe frequency-based method with exponential decaying [13] that is applied hereinafter. The relative values of measured signals $u(t), y(t)$ are computed first

$$\begin{aligned} \Delta u(t) &:= u(t) - u_0, \\ \Delta y(t) &:= y(t) - y_0 \end{aligned} \tag{1}$$

where $\{u_0, y_0\}$ expresses the operating point. Then, $\Delta u(t), \Delta y(t)$ are subject to the decay

$$\begin{aligned} u_a(t) &= \Delta u(t) \exp(-at), \\ y_a(t) &= \Delta y(t) \exp(-at) \end{aligned} \tag{2}$$

for some $a > 0$, so that absolute values of $u_a(t_{fin}), y_a(t_{fin})$ are sufficiently small for the final measurement time t_{fin}. The following formula holds from the Fourier transform

$$G(\mathrm{j}\omega_l + a) = \frac{Y(\mathrm{j}\omega_l + a)}{U(\mathrm{j}\omega_l + a)} = \frac{\int_0^{t_f} y_a(t) \exp(-\mathrm{j}\omega_l) \mathrm{d}t}{\int_0^{t_f} u_a(t) \exp(-\mathrm{j}\omega_l) \mathrm{d}t} \tag{3}$$

where $G(s)$ expresses the process transfer function and $U(s), Y(s)$ means input and output Laplace images, respectively. By matching (3) with the process model transfer function $G_m(s)$, a set of algebraic equations can be obtained for particular (selected) angular frequencies $\omega_l \geq 0$. Note that it is originally suggested to be taken as

$$\begin{aligned} \omega_l &= l \frac{2\pi}{t_{fin} + T_s}, \\ l &= 1, 2, \ldots, 0.5\left(t_{fin} / T_s + 1\right) \end{aligned} \tag{4}$$

where t_{fin} is assumed to be an integer multiple of the sampling period T_s.

The process static gain K can be estimated either from (3) for $\omega_l = 0$ or via

$$K = \frac{\int_0^{T_{osc}} \Delta y(t + \tau) \mathrm{d}\tau}{\int_0^{T_{osc}} \Delta u(t + \tau) \mathrm{d}\tau}, \tag{5}$$

which simply means the ratio of shifted input and output signal surfaces within the period. A relay with an asymmetric static characteristic is used to avoid the cancellation of positive and negative areas in (5). Let us consider the on/off relay (without hysteresis, for simplicity) governed by the characteristics as in Fig. 2 where $B^+ \neq \left|B^-\right|$, $B^+ > 0, B^- < 0$.

III. MOTIVATION AND MAIN RESULT

It is evident from (1)-(3) and (5) that the knowledge of the "true" operating point $\{u_0, y_0\}$ is crucial for a correct model parameters estimation. Let $u_0 = 0.5\left(B^+ + B^-\right)$ be fixed. Then, the research question is how to estimate y_0 whenever $y_0 \neq r$ due to relay and process asymmetry, see Fig. 3 [8]. Note that the meaning of y_m will be introduced later. Consider (without the loss of generality) that $r = 0$ hereinafter.

A. *Research Question Motivation*

Example 1 demonstrates the invalidity of (5) for different settings of B^+, B^- if equality $y_0 = r$ is assumed.

Example 1. Let the process be governed by ideal transfer functions

$$\begin{aligned} G^+(s) &= \frac{30}{s^4 + 8s^3 + 24s^2 + 32s + 15}, \\ G^-(s) &= \frac{200}{s^4 + 12s^3 + 54s^2 + 108s + 80} \end{aligned} \tag{6}$$

where $G^+(s)$ and $G^-(s)$ express the dynamics if $u(t) = B^+$ and $u(t) = B^-$, respectively [8].

For $y_0 = r$, values of K computed using (5) are displayed in TABLE I. It is obvious from the table that the higher B^+ is, the better estimation of $G^+(0) = 2$ is.

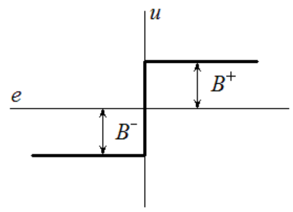

Fig. 2. Asymmetric on/off relay (without hysteresis) static characteristics.

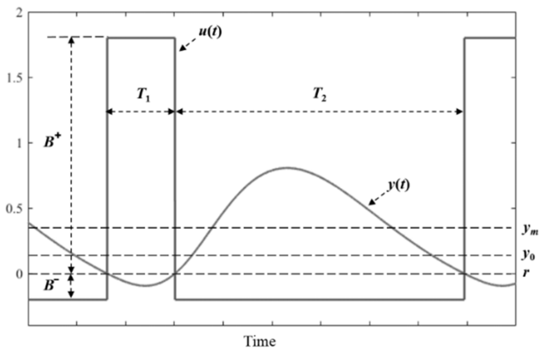

Fig. 3. Sustained oscillations with relay and process asymmetry [8].

TABLE I. COMPUTED STATIC GAINS – EXAMPLE 1

B^+	B^-	K	y_0
0.2	-1.8	3.0747	-0.0829
0.5	-1.5	4.9184	-0.1787
0.8	-1.2	-9.6217	-0.2440
1	-1	-0.8521	-0.2289/ -0.2690[a]
1.2	-0.8	0.4013	-0.2133
1.5	-0.5	1.1500	-0.1639
1.8	-0.2	1.6205	-0.0794

a. For G^+/G^-.

Contrariwise, the lower the value of B^+ is selected, the closer value of K to $G^-(0)=2.5$ is. However, the computational results fail completely if $B^+ \approx |B^-|$; i.e., the relay is symmetric or almost symmetric. It implies the intuitive idea that the setting $B^+ \gg |B^-|$ can serve for parameter identification of submodel $G_m^+(s)$ while $B^+ \ll |B^-|$ can be used for $G_m^-(s)$. Nevertheless, one has to be careful about the existence of sustained oscillations for a high value of B^+.

Simultaneously, it is expected that the better estimation also means that "true" y_0 is closer to r. The rightmost column of TABLE I. hence, provides the reader with values of y_0 for which the static gains K equal actual $G^+(0)$ or $G^-(0)$ when $B^+ \geq |B^-|$ or $B^+ \leq |B^-|$, respectively.

It is expected that the same principle also holds for the model parameters determining its dynamics. However, a proposition of a reasonable guess of y_0 remains unsolved.

B. The Iterative Procedure Design

We start the idea of the operating point computation by observing the value of y_m (see Fig. 3), which expresses the average value of $y(t)$ amplitudes (or a DC shift of the process output). From (5), one can conclude that

$$K \int_0^{T_{osc}} \Delta u(t+\tau)\,\mathrm{d}\tau = \underbrace{\int_0^{T_{osc}} \Delta y(t+\tau)\,\mathrm{d}\tau}_{:=y_{INT}(y_0)}$$
$$K\underbrace{\left(T_1 B^+ + T_2 B^-\right)}_{:=u_{INT}} = y_{INT}(y_0). \tag{7}$$

If $r = y_0$, the surface y_{INT} can also be viewed as a rectangle of the area equal to the integral in (7)

$$y_{INT}(y_0) = y_m T_{osc} = y_m (T_1 + T_2). \tag{8}$$

However, whenever $r \neq y_0$, formula (8) does not hold true as the offset y_m must be considered relatively to (a nonzero) y_0. Then, relation (8) changes to

$$y_{INT}(y_0) = (y_m - y_0) T_{osc} = (y_m - y_0)(T_1 + T_2). \tag{9}$$

By combining (7) and (8) (or (9)), two possible estimates of y_m are obtained

$$\begin{aligned} {}^1y_m &= K\frac{u_{INT}}{T_{osc}}, \\ {}^2y_m(y_0) &= \frac{y_{INT}(y_0)}{T_{osc}} \end{aligned}. \tag{10}$$

Remark 1. The left-hand side of (10) enables calculation of K with less computational effort than via (5) for symmetric systems since it does not require the knowledge of y_{INT}.

Hence, the goal is to find y_0 for which ${}^1y_m = {}^2y_m$. Assume that K is known exactly for some current (*k*th iteration) operating point estimation y_0^k. If it is computed that ${}^1y_m < {}^2y_m(y_0^k)$, i.e.,

$${}^1y_m + \Delta y_m = {}^2y_m(y_0^k),\ \Delta y_m > 0 \tag{11}$$

at this point, the value of $y_{INT}(y_0)$ should be less in fact. Therefore, a new estimate must satisfy $y_0^{k+1} > y_0^k$, i.e.,

$$y_0^{k+1} = y_0^k + \Delta y_0^k,\ \Delta y_0^k > 0, \tag{12}$$

see (9). And vice versa. The simple assumption $\Delta y_m = \Delta y_0^k$ yields the eventual iterative formula from (11) and (12)

$$y_0^{k+1} = y_0^k + {}^2y_m(y_0^k) - {}^1y_m. \tag{13}$$

Now, the following iterative procedure can be assembled:

Step 1. Perform the relay-feedback experiment with $B^+ > \left|B^-\right|$ for $G_m^+(s)$ and/or $B^+ < \left|B^-\right|$ for $G_m^-(s)$, and save the data.

Step 2. Set $y_0 = r = 0$ and select $a > 0$. Select the initial submodel parameter set (e.g., randomly).

Step 3. Compute $\hat{K}$ via (5) to get the current estimate of submodel parameters relation for $\omega = 0$.

Step 4. Calculate (3) for a necessary number of angular frequencies $\omega_l > 0$ (e.g., as in (4)) and solve the obtained set of algebraic equations along with the static gain guess from Step 3 to get the complete submodel parameter set.

Step 5. Based on the obtained parameter set, calculate the updated submodel static gain $\hat{K}$.

Step 6. Compute (10) and update the estimation of $\hat{y}_0$ via (13), and go to Step 3.

Example 2. Assume again the asymmetric process with subsystems' dynamics given by (6). Based on TABLE I. , let the setting of the simple asymmetric on/off relay be $B^+ / B^- = 1.8/-0.2$ and $B^+ / B^- = 0.2/-1.8$ to identify submodels $G_m^+(s)$ and $G_m^-(s)$, respectively. Set $r = 0$ as default. Simulated process inputs and outputs that transit to sustained oscillations with periods $T_{osc}^+ = 3.313$ s and $T_{osc}^- = 2.514$ s are displayed in Fig. 4.

Consider the submodels of order 4 as well, for simplicity

$$G_m(s) = \frac{b_0}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}. \quad (14)$$

Initial static gain guesses $\hat{K}^0$ are given in TABLE I. and select $\omega_l, l = \overline{1,20}$, according to (4). Let us denote by $\hat{\mathbf{g}} := \left[G(\mathrm{j}\omega_l + a)\right]$ frequency point vectors computed via (3) and by $\mathbf{g}_m(\mathbf{p}) := \left[G_m(\mathbf{p}, \mathrm{j}\omega_l + a)\right]$ those of (14) with parameter set $\mathbf{p} = [b_0, a_3, a_2, a_1, a_0]$ to be identified. We attempt to solve the following optimization problem in each iteration step

$$\begin{gathered} \mathbf{p}^* = \arg\min_{\mathbf{p}} F(\mathbf{p}) \\ F(\mathbf{p}) := \left\| \begin{bmatrix} \hat{K} \\ \hat{\mathbf{g}} \end{bmatrix} - \begin{bmatrix} b_0 / a_0 \\ \mathbf{g}_m(\mathbf{p}) \end{bmatrix} \right\|_2 + \alpha \Pi(\mathbf{p}) \end{gathered} \quad (15)$$

where $\alpha > 0$ and $\Pi(\mathbf{p}) = -\sum_{i=2}^{5} \log\left(1 - \exp(p_i)\right)$ represents a penalty function (in which p_i denotes the ith entry of $\mathbf{p}$) meaning the necessary stability condition of the model.

Select the well-established variable-simplex optimization algorithm [14] is used to tackle with (15).

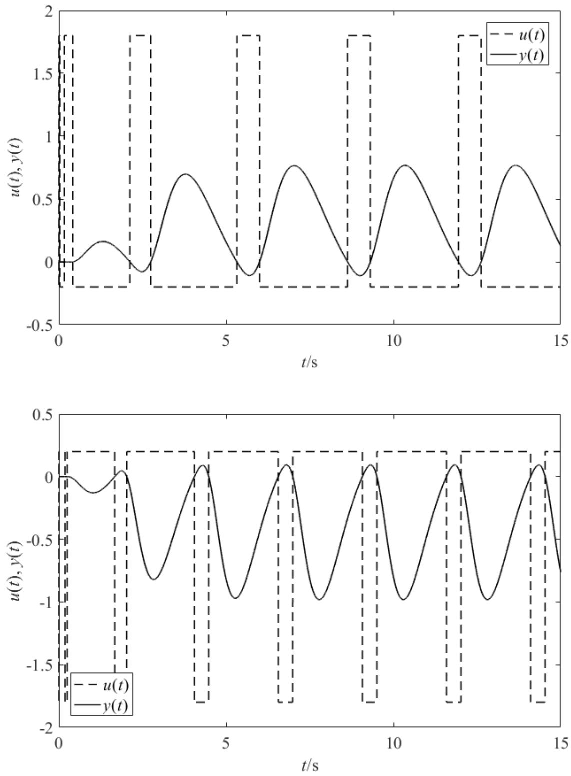

Fig. 4. Sustained oscillations for $B^+ / B^- = 1.8/-0.2$ (up) and $B^+ / B^- = 0.2/-1.8$ (down) – Example 2.

Let the initial parameters' estimation be ${}^1\mathbf{p}^0 = \left[2\hat{K}^0, 2, 4, 4, 2\right]$ with remaining simplex points ${}^i\mathbf{p}^0 = {}^1\mathbf{p}^0 + \beta \mathbf{e}_{i-1}, i = \overline{2,6}$ where $\mathbf{e}_i$ is the Euclidean unit vector with 1 at the ith position.

Courses of $\hat{K}$ and $\hat{y}_0$ during iterations for selected settings of a, α, β, and for $G_m^+(s)$ are displayed in Fig. 5. Analogously, those results for $G_m^-(s)$ are provided in Fig. 6. Note that data already given in TABLE I. are omitted, and the solid lines mean the ideal values in the figures.

Eventually, the selected final identified model parameter values are given in TABLE II. Fig. 7 displays process and model step responses. Besides the optimized model for fixed setting $y_0 = r = 0$ is added to the comparison. Analogously, Nyquist plots are benchmarked in Fig. 8.

It is worth noting that the proposed iterative procedure provides better time-domain and frequency-domain performance measures compared to the assumption that $y_0 = r = 0$ and also against some other relay-based identification techniques (based on the frequency-fitting principles and even those using the describing function). The reader is referred to [8] for more detail.

We have found during numerical tests that a better static gain estimation does not necessarily coincides with a better model dynamics' parameters identification. In other words, the best spectral matching can be found for a different operation point estimation than the ideal one (from the static gain viewpoint).

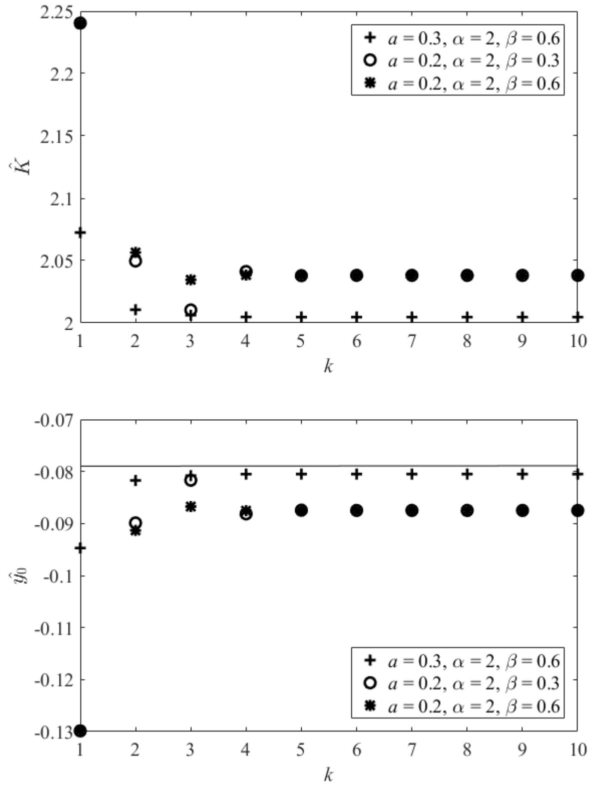

Fig. 5. Iterated values of $\hat{K}$ (up) and $\hat{y}_0$ (down) for $G_m^+(s)$ – Example 2.

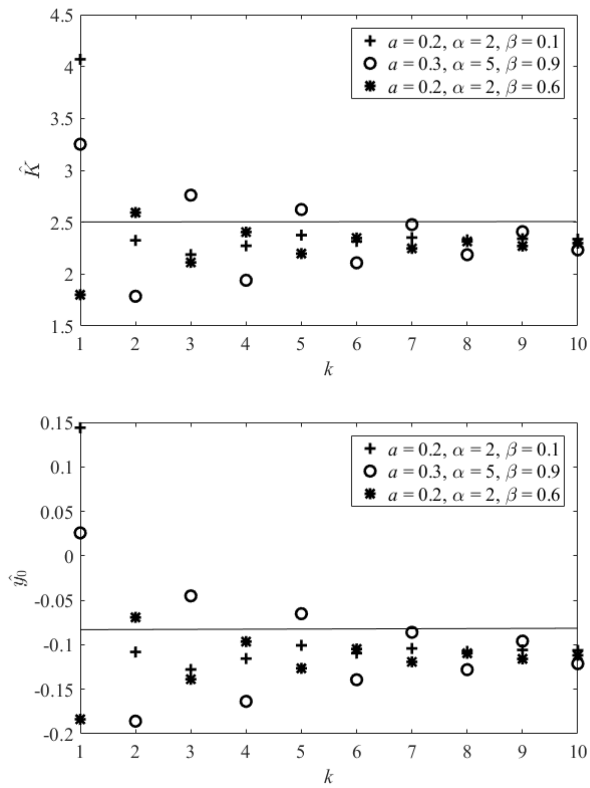

Fig. 6. Iterated values of $\hat{K}$ (up) and $\hat{y}_0$ (down) for $G_m^-(s)$ – Example 2.

TABLE II. EVENTUAL SUBMODEL PARAMETERS – EXAMPLE 2

Submodel	b_0	a_3	a_2	a_1	a_0
$G_m^+(s)$	31.9219	7.6641	25.9764	41.8680	15.6603
$G_m^-(s)$ [x 10^4]	14.7905	0.5750	3.0530	5.7864	6.3409

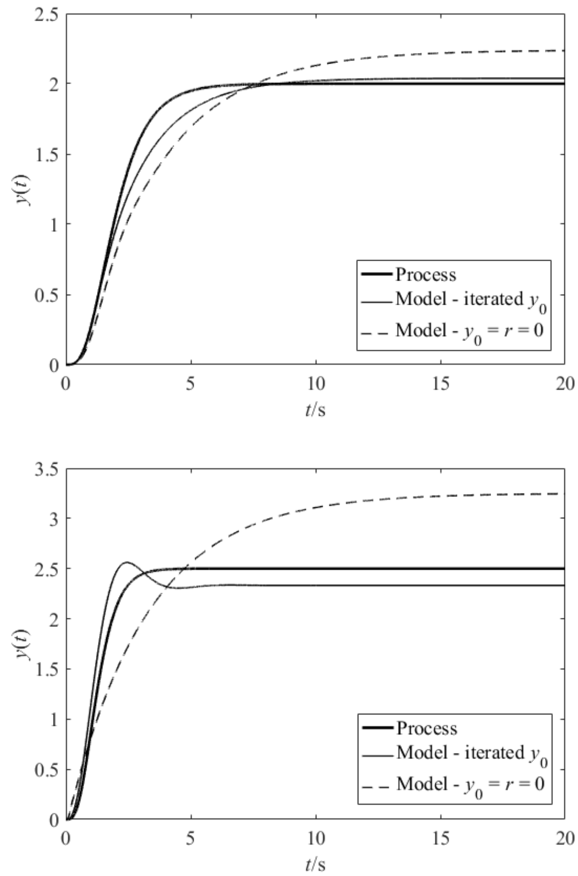

Fig. 7. Step responses comparison for $G_m^+(s)$ (up) and $G_m^-(s)$ (down) – Example 2.

IV. DISCUSSION AND FURTHER RESEARCH SUGGESTIONS

The proposed strategy suffers from many discrepancies and requires further improvements, mainly regarding numerical issues. A list of some possible enhancements and open problems follows.

A different frequency set from (4) might be considered. Although some authors postulate that angular frequencies higher than the so-called ultimate one ($\approx \omega_{osc}$) cannot be considered [7], it is not reasonable from the mathematical point of view. The only theoretical limitation is half of the angular sampling frequency ($2\pi / T_s$).

Another crucial task is the setting of a in (3). We have observed via numerical tests that the best-optimized results (for iterated $\hat{y}_0$) could be obtained for a different value of a than the one that gives the best estimation with ideal y_0. In [15], the authors recommend to take the setting $a = \omega_{osc} / 20$.

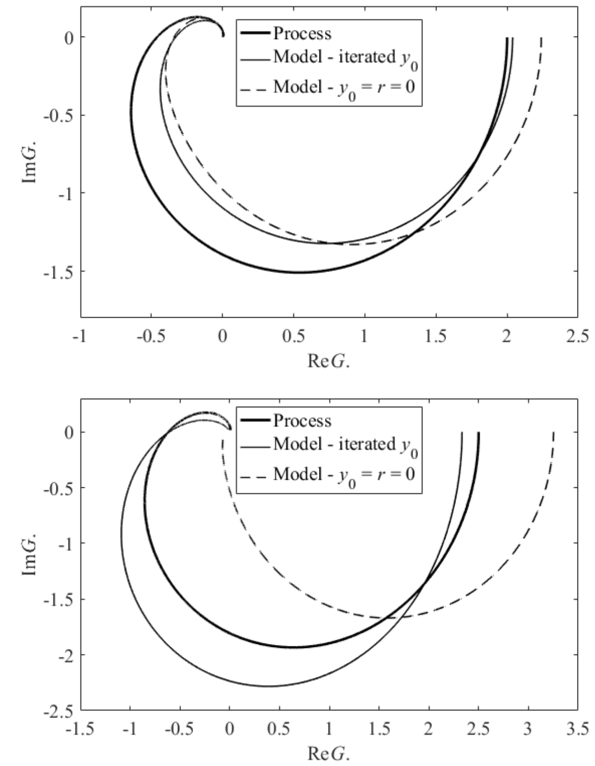

Fig. 8. Nyquist plots comparison for $G_m^+(s)$ (up) and $G_m^-(s)$ (down) – Example 2.

The decaying can also be applied not to the whole time interval $t \in [0, t_{fin}]$ but within the sustained oscillation period only. More precisely, it holds for a periodic signal that

$$G(\mathrm{j}\omega + a) = \frac{\dfrac{\int_0^{T_{osc}} y_a(t+\tau)\exp(-\mathrm{j}\omega)\,\mathrm{d}\tau}{1-\exp(-(\mathrm{j}\omega + a)T_{osc})}}{\dfrac{\int_0^{T_{osc}} u_a(t+\tau)\exp(-\mathrm{j}\omega)\,\mathrm{d}\tau}{1-\exp(-(\mathrm{j}\omega + a)T_{osc})}} = \frac{\int_0^{T_{osc}} y_a(t+\tau)\exp(-\mathrm{j}\omega)\,\mathrm{d}\tau}{\int_0^{T_{osc}} u_a(t+\tau)\exp(-\mathrm{j}\omega)\,\mathrm{d}\tau}, \tag{16}$$

i.e., formula (4) represents a special case of (16) for $a = \omega = 0$. It is, moreover, questionable whether a setting $\omega \neq 0$ may give a good result.

The presented framework idea can also be naturally used for other relay-based identification methods (i.e., not only for [13]), which suggests attempting to benchmark selected ones.

We have also observed that some settings of the triplet a, α, β do not yield solution convergence. For these cases, one may either find the result with the lowest fitness within the set of all iterative steps or reset the estimation of $\mathbf{p}$ in each step (yet preserve the update of $\hat{y}_0$).

Many research questions can also be raised regarding the definition of the cost function and constraints. A selection of suitable optimization techniques goes hand in hand with that.

Last but not least, it would be challenging to identify parameters of models with more complex dynamics (e.g., infinite-dimensional ones [16]) and verify the proposed procedure via a real-time laboratory experiment.

V. Conclusions

The presented preliminary research has been induced by the observation that the use of asymmetric (and even symmetric) relay when identifying systems and processes with possible asymmetric dynamics yields a mismatch between the reference signal and the operating point. The crucial step of the proposed iterative procedure (including a subproblem optimization) has been to guess the actual operating point value. Numerical results have proven the reasonability of the proposed strategy.

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